Stratified Flow Model for Two-Phase Pressure Drop Prediction in Trickle Beds

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The steady downward flow of a gas and a liquid in a packed bed for low superficial velocities of both phases is termed the trickle flow regime. Trickle-flow reactors are used in a variety of practical chemical and biochemical applications.

The most important macroscopic hydrodynamic parameters for trickling flows in packed beds are the pressure drop and liquid holdup. The current level of understanding of the hydrodynamics of these systems allows a prediction of these parameters by means of empirical correlations; a number of reviews exist on this subject (Satterfield, 1975; Charpentier, 1976; Hofmann, 1977; Shah et al., 1978; Gianetto et al., 1978; Shah, 1979; Van Landeghem, 1980; Herskowitz and Smith, 1983). In recent times, the catalyst dimensions in trickle-flow reactors have been reduced. The advantages of this include:

- Decrease in backmixing
- Increase in wetted fraction
- Increase in mass transfer coefficients
- Decreased maldistribution
- Increased effectiveness of catalysts

Achieving a high effectiveness factor is of particular importance for a range of biological reactions where the cost of the immobilized protein is high.

Despite these advantages, there have been limited studies on holdup and pressure drop characteristics of beds with small packings, less than about 2 mm in size. Kan and Greenfield (1979), who have conducted studies with such supports, showed that existing empirical correlations were inadequate for hydrodynamic predictions for small packings, due to the increased importance of surface tension effects.

To enhance the understanding of trickle-flow reactors containing small packings, theoretical studies are needed which examine the hydrodynamics of the flow at the pore level, thus adding a scientific basis to any hydrodynamic models that are developed. Levec et al. (1986) attempted this for packings in the size range 3–6 mm, where they modeled the packed bed as a number of parallel conduits in which a two-phase annular flow structure was assumed. In the work reported in this note, the stratified downflow structure, Figure 1, which has been shown to occur in capillary tubes of less than 1 mm in size, has been used as a basis for a new model for trickle-bed operation with packings less than 1 mm in size. Surface tension forces, which are thought to determine the hydrodynamics of flow in trickle-flow reactors with small packings, stabilize the stratified flow structures in capillaries of small internal diameter (<1 mm).

Model Development

Consider a bed packed with small particles (<1 mm), in which the stratified flow model developed by Biswas and Greenfield (1985) is valid. Initially the bed is saturated with the wetting phase (e.g., water). As a nonwetting phase (e.g., gas) is introduced, it will gradually replace the existing wetting phase. However, there will be an amount of the wetting phase clinging to the particles that is immobile. If this immobile wetting phase is assumed to form tortuous flow channels, then each flow channel can be visualized as a capillary tube (Vuckovic, 1976; Kan and Greenfield, 1978; Biswas, 1982; Saez et al., 1986) of diameter d_i and length L_i. For a porous system of bulk volume V_r and effective porosity ϕ and a volume equivalent saturation of d_s (fraction of porous system that is accessible to the mobile phase), the number of capillaries n_i can be calculated as:

$$n_i = \frac{4V_r}{\pi d_i^2 L_i} \phi\, d_s \qquad (1)$$

Use of Eq. 1 allows the number of capillary tubes, each of diameter d_i, in the bundle simulating the network to be estimated from a capillary pressure test (Vuckovic, 1976; Dullien, 1979).

In addition to the assumptions stated in the derivation of the stratified flow model for vertical capillaries (Biswas and Greenfield, 1985), the following assumptions must be made to extend

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the model to a packed bed:

1. Equilibrium flow only occurs in the conductive pore volume (reduced network of pore channels)

2. Entrance and exit pressures are identical for each reduced flow channel

3. With uniform entrance and exit pressures the gas saturation in all reduced pore channels must be equal. The saturation of fluid phase i is defined as

$$S_i = \frac{\text{Volume of fluid phase } i \text{ in the sample}}{\text{Total accessible pore volume in the sample}} \tag{2}$$

4. The bundle of capillary tubes pore structure model is valid.

5. The tortuosity of the bed is determined by the interstitial saturation of the immobile phase. The cohesive forces causing the interstitial saturation between the particles are far stronger than those resulting from gas-liquid shear, that is, tortuosity is independent of gas-liquid flow rates. This will be true only over a limited range of flow rates.

For single-phase flow of a nonwetting phase through a bed containing an interstitial saturation of a wetting phase, Darcy's law states:

$$Q' = \frac{K' A_r \Delta P}{\mu_{nw} L} \tag{3}$$

where K' is the reduced permeability, A_r the cross-sectional area, and L the length of the bed. The absolute permeability, K, is given by the same equation applied to a dry bed. By combining the Poiseuille equation (Bird et al., 1960) and Eq. 1 for all capillaries of diameter d_i and summing over the pore size distribution, the following equation results:

$$Q' = \frac{V_r \phi \Delta P}{32 \mu_{nw} L_e^2} \int d_i^2 \, dS \tag{4}$$

where $\int d_i^2 \, dS$ is derived from the pore size distribution and L_e is the effective length of the capillaries, assuming all to have equal length. From Eqs. 3 and 4 the tortuosity can be defined as:

$$\tau = \frac{L_e}{L} = \left(\frac{\phi \int d_i^2 \, dS}{32\, K'} \right)^{0.5} \tag{5}$$

For two-phase flow through a single capillary, the fluid distribution at any cross section, for any ratio of fluid rates, can be depicted as shown in Figure 1. Circle *1* represents the capillary wall while circle *2* represents the fluid-fluid interface. If each flow channel is considered as a separate, noncircular capillary inside a circular capillary of diameter d_i, then the hydraulic diameter of each channel is given by:

$$D_{hi} = \frac{4 \times \text{flow area of the phase } i}{\text{phase } i \text{ channel perimeter}} \tag{6}$$

where i can be either a wetting or a nonwetting phase.

If r and R represent the radii of the fluid-fluid interface (circle *2* of Figure 1) and the capillary (circle *1* of Figure 1), respectively, then the channel perimeter for the wetting phase is given by:

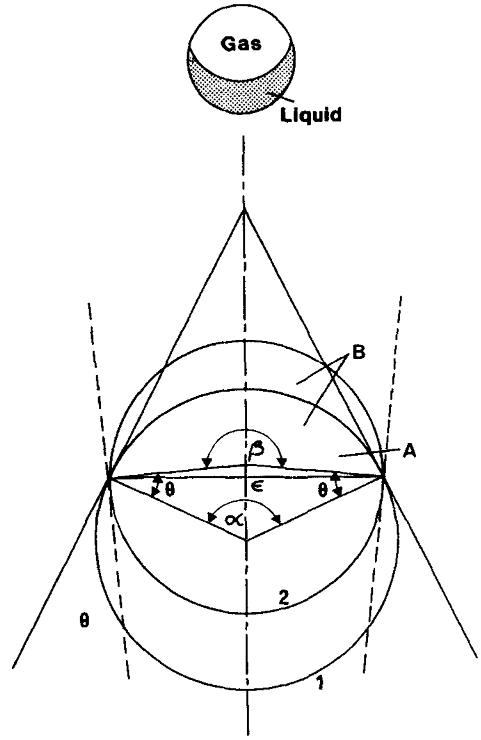

Figure 1. Geometric relationship for wetting and nonwetting fluid flow channels in a capillary.
After Biswas and Greenfield (1985)

$$C_W = 2\pi R - 2\pi R(\alpha/360) + 2\pi r - 2\pi r(\beta/360) \tag{7}$$

The terms α, β represent the angles subtended by chord ϵ in circle *1* and circle *2*, respectively. Since $\beta = \alpha + 2\theta$ and $r = Rx/y$ [where $x = 2\sin(\alpha/2)$ and $y = 2\sin(\beta/2)$], the channel perimeter for the wetting phase can be written as:

$$C_W = 2R(1 - M) \tag{8}$$

where

$$M = \left[\frac{\alpha}{360} \left(1 + \frac{x}{y} \right) - \frac{x}{y} \left(1 - \frac{2\theta}{360} \right) \right] \tag{9}$$

Similarly, the channel perimeter for the nonwetting phase is,

$$C_{nw} = 2\pi R N \tag{10}$$

where

$$N = \left[\frac{\alpha}{360} \left(1 - \frac{x}{y} \right) + \frac{x}{y} \left(1 - \frac{2\theta}{360} \right) \right] \tag{11}$$

By using Eqs. 8 and 10, the flow rates of the wetting and the nonwetting phases through a single capillary are found to be (Biswas and Greenfield, 1985):

$$q_w = \frac{d_i^4 S_w^3 \pi \Delta P}{128(1 - M)^2 \mu_w L} \tag{12}$$

and

$$q_{nw} = \frac{d_i^4 S_{nw}^3 \pi \Delta P}{128 N^2 \mu_{nw} L} \tag{13}$$

Hence the flow rates of the wetting and the nonwetting phases through the packed bed, using Eq. 5, are given by:

$$Q_w = \Sigma q_w n_i = \frac{S_w^3}{(1-M)^2} \frac{V_r \Delta P K'}{\mu_w L^2} \tag{14}$$

$$Q_{nw} = \Sigma q_{nw} n_i = \frac{S_{nw}^3}{N^2} \frac{V_r \Delta P K'}{\mu_{nw} L^2} \tag{15}$$

Taking the ratio of the flow rates of the wetting and the nonwetting phases, Eqs. 14 and 15, and using the relation $S_{nw} = 1 - S_w$, we obtain

$$\frac{Q_w}{Q_{nw}} = \frac{S_w}{(1-S_w)^3} \frac{N^2}{(1-M)^2} \frac{\mu_{nw}}{\mu_w} \tag{16}$$

From Figure 1 S_w can be estimated to be

$$S_w = \left[1 - \frac{\frac{1}{2}\left(\frac{\pi\alpha}{180} - \sin\alpha\right)}{\pi}\right] - \frac{x^2}{y^2}\left[1 - \frac{\frac{1}{2}\left(\frac{\pi\beta}{180} - \sin\beta\right)}{\pi}\right] \tag{17}$$

Equations 16 and 17 are solved simultaneously, using a numerical technique, to obtain S_w and the central angle α. The bed pressure drop is then calculated using either Eq. 14 or Eq. 15.

To predict the pressure dop in a trickle-flow reactor with small packings, the following parameters must be known *a priori:*

1. The reduced permeability K', determined from single-phase flow experiments; see Figure 2
2. Wetting and nonwetting phase viscosities μ_w and μ_{nw}

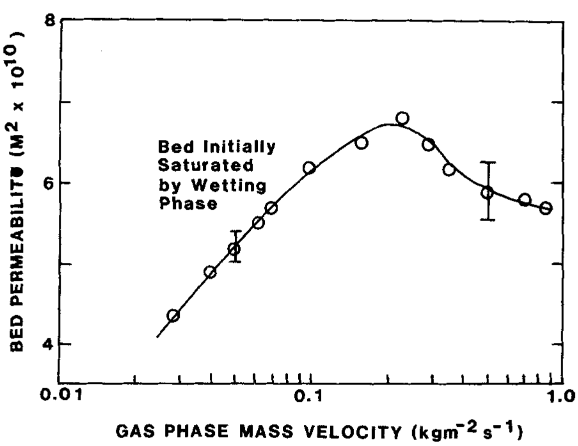

Figure 2. Reduced permeability as a function of gas phase mass velocity in bed of 1.0 mm sperical glass packings.
$V_r = 197$ cm³; $\phi = 0.34$

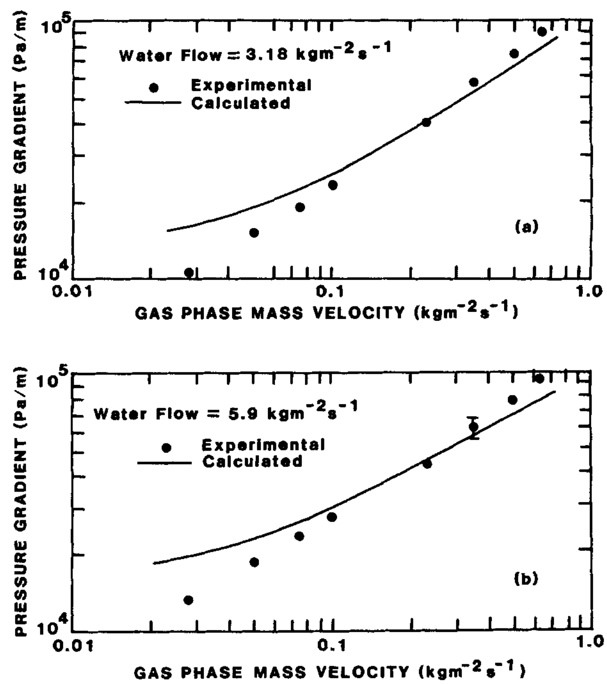

Figure 3. Test of pressure drop model for air-water system in a bed of 1.0 mm spherical glass packings.
$V_r = 197$ cm³; $\theta = 40 \pm 5$ degrees; $\phi = 0.34$

3. Apparent wettability angle θ (Biswas and Greenfield, 1985)
4. Bed voidage ϕ and volume V_r

Results and Discussion

The experimental apparatus of Kan and Greenfield (1979) was used in these trickle-bed studies with 1 mm nonporous glass packings. Predictions, using Eq. 3, of reduced permeability K' for a bed containing an interstitial saturation of liquid are shown in Figure 2. This value defines the tortuosity of the packed bed at any value of gas flow rate. The results from applying the overall model for the prediction of two-phase pressure drop are shown in Figure 3. The model gives a root mean square error of less than 10% for gas flow rates in the range 0.1–1 kg/m²s.

In conclusion, a fundamental two-phase conduit model, which assumes a stratified flow structure, has been developed and tested for trickle flow reactors with packings less than 1 mm in size. The prediction of pressure drop is excellent in the range 0.1–1 kg/m²s for a number of liquid flow rates. This model, which inherently accounts for surface tension effects, gives a more fundamental understanding of flows in porous media.

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